Analogies Between Gravity and Fluid Dynamics

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Abstract

Gravity is much more difficult to combine with quantum mechanics than any other force. In recent decades, some analogies between gravity and thermodynamics, and hydrodynamics suggest that gravitation is an emergent phenomenon. We propose here to give a physical meaning to the analogy between gravity and hydrodynamics in particular. From two models: absorption and thermal, we find some aspects of gravitation in the Newtonian limit. We speculate the link with thermodynamics derived from the analogy between gravity and fluid dynamics. The relativistic description of our model is inspired by the Einstein-æther theory, which is a generally covariant theory of gravity containing a dynamical preferred frame.

Declaration

I, the undersigned, hereby declare that the work contained in this essay is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Yvan Jordan Ngucho Mbeutchou, 31 May 2019.

Conventions, notations, and acronyms

Units. Numerical results are mostly given in terms of units of the International System. In abstract calculations, we adopt the usual relativistic convention of $c = 1$, so that lengths and times have the same dimension.

Differential geometry. The signature of spacetime's metric is taken to be $(-+++)$. We use Einstein's summation rule over repeated indices, the range of the sum being dictated by the nature of the indices (see Table 1).

Notations. See Table 1 for a list of the recurrent symbols used in this dissertation. As often in the relativity literature, vectors and tensors are identified with their components over an arbitrary coordinate basis (e.g. $\mathbf{u}$ will be equivalently denoted u^a). Partial derivatives with respect to, e.g. coordinate x^a are abridged as

$$\frac{\partial f}{\partial x^a} \equiv \partial_a f \equiv f_{,a}, \tag{0.0.1}$$

for any function or tensor f.

Acronyms. They will be defined when used for the first time. See also Table 2

Notation	Description
$\equiv$	definition
lowercase latin indices $a, b, \cdots, h$	run from 0 to 3
lowercase latin indices $i, j, \cdots$	run from 1 to 3
$\mathrm{d}S = r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$	infinitesimal surfaces in spherical coordinates
$\mathbf{u}$, u^a	four-velocity
$\mathbf{v}$, v^a	four-velocity of æther or for short æther
$\vec{v}_{\text{ae}}$	velocity of æther in the Newtonian level
τ	proper time
η_{ab}, η	Minkowski metric, determinant of the metric
$\rho_{æ}$, ρ	æther density, matter density
δ_D	Dirac distribution
$\gamma = -v_a u^a$	Lorentz factor of particles in æther frame
G, Φ	gravitational constant, gravitational potential

Table 1: Description of the main notations used in this Essay.

Acronym	Signification
QFT	Quantum Field Theory
GR	General Relativity
æ-theory	Einstein-æther theory
AdS/CFT	Anti De Sitter space/Conformal Field theory
LIGO	Laser Interferometer Gravitational-Wave Observatory
PDE	Partial Differential Equation
SR, STR	Special Relativity, Special Theory of Relativity

Table 2: List of acronyms and their signification.

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1. Introduction

The Universe is currently described by two remarkably effective major theories: Einstein's general relativity, portraying the Universe at the macroscopic scale by providing a description of gravitation and quantum mechanics depicting the Universe at the microscopic scale by providing an explanation of the other three elementary interactions, namely: electromagnetic, weak nuclear and strong nuclear interactions. However, some phenomena in the Universe require the simultaneous application of both theories and therefore require the development of a theory of quantum gravity.

In 1915, Albert Einstein published his work on general relativity [5]. The latter indicates that any energy source causes a local deformation of space-time, so the gravitation is the manifestation of curvature of space-time. In his theory, Einstein treats space-time as a pseudo-Riemannian manifold, i.e. a smooth space with dynamical geometry. Technically, within Einstein's equation, the energy-momentum tensor is linked to the deformation of space-time.

At the same time, in the beginning of the 20th century, quantum mechanics appeared. Its mission is to describe the interactions between physical systems and their dynamics on a quantum scale. The theoretical advancements in quantum mechanics led to the development of quantum field theory (QFT), thanks to the second quantization provided by Paul Dirac in 1928. QFT describes particles and their interactions in terms of quantum fields and today forms the physico-mathematical basis of the Standard Model of particle physics. The latter thus formalizes the electromagnetic, weak and strong nuclear interactions within quantum theories such as quantum electrodynamics, electroweak theory or quantum chromodynamics.

Since their formulation, these two theories have not ceased to be tested and are now remarkably well supported by explaining in a coherent and elegant way the phenomena belonging to their respective fields. We can cite the proof that Neutrino has mass in 1998, the discovery of the Higgs boson on March 2013. Also, the discovery of the gravitational waves by the LIGO and Virgo Scientific Collaboration on February 2016 and most currently the first-ever picture of a black hole in the galaxy M87. Nevertheless, despite this amazing descriptive and predictive ability, they do not cover all the phenomena present in the Universe. This is particularly true for the interior of black holes and the early Universe.

When general relativity is concerned with describing the interior of black holes, theoretical anomalies emerge in the form of gravitational singularities. The latter, unlike what is often read do not correspond to tangible objects but to a mathematical limit of Einstein's theory, a kind of "pathology" of general relativity correlated to the existence of infinite quantities. A singularity appears when the theory is unable to describe such extreme gravitational phenomena. The same is true for the beginning of the Universe; Einstein's equations indicate that at the time of the Big Bang, the Universe was reduced to an initial singularity.

The existence of these anomalies indicates an incompleteness of general relativity. At such energy and density levels, quantum effects emerge within the gravitational field; quantum mechanics must therefore be involved in describing these events. More precisely, these quantum gravitational effects would emerge at the Planck scale. Indeed, the Planck length is given by the following formula:

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \,, \tag{1.0.1}$$

where "$\hbar$" denotes the reduced Planck constant, "G" the gravitational constant and "c" denotes the speed of light in vacuum. This formula combines a quantum constant and a gravitational constant, which suggests at the emergence of quantum gravity on a Planck scale. A theory of quantum gravity, unifying quantum mechanics and general relativity, is therefore necessary in order to describe these phenomena.

However, the unification of the two theories is extremely complex in view of the significant differences between them. Quantum field theory describes the elementary interactions within a flat space-time, i.e. the space-time of special relativity corresponding to the Minkowski metric. While in general relativity, gravitation is encoded in a curved space-time corresponding to a pseudo-Remannian metric. Moreover, in Einstein's theory space and time are intimately intertwined, whereas in quantum mechanics time is not a physical quantity, no operator can be associated with it.

So gravity is considerably harder to combine with quantum mechanics than all the other forces. The quest for unification of gravity with these other forces of Nature at a microscopic level, may therefore not be the right approach. It is known to lead to many problems, paradoxes and puzzles. String theory has solved some, but not all of these problems to some extent and we still have to find out what the theoretical solution of string theory teaches us.

It has been suggested that gravity and space-time geometry could be emergent. Also, string theory and its related developments have given several indications in this direction. Particularly, important clues come from the AdS/CFT correspondence (Anti De Sitter space/Conformal Field theory), or more generally, the open/closed string correspondence. This correspondence leads to a duality between theories that contain gravity and those that do not. It suggests that gravity can emerge from a microscopic description that does not know about its existence. We also have to note that if the emergence of gravity is true, that means that the quest of quantum gravity will not be achieved by looking for the quantization of gravity, rather the quantization of the microscopic degrees of freedom that generate gravity.

The universality of gravity suggests that its emergence should be understood from general principles that are independent of the specific details of the underlying microscopic theory. It is also demonstrated by the fact that its basic equations closely resemble to the laws of thermodynamics and hydrodynamics.

It has been suggested since the 1990s by Ted Jacobson [9] that, the Einstein's equation is derived from the proportionality of entropy and horizon area together with the fundamental relation $\delta Q = T\mathrm{d}S$ connecting heat, entropy, and temperature. Viewed in this way, the Einstein's equation is an equation of state. This perspective suggests that "it may be no more appropriate to canonically quantize the Einstein's equation than it would be to quantize the wave equation for sound in air". This approach has been more developed by Jarmo Mäkelä and Ari Peltola [12] and recently, an elegant approach of Erik Verlinde [17] where starting from first principles and general assumptions Newton's law of gravitation is shown to arise naturally and unavoidably in a theory in which space is emergent through a holographic scenario. From The equivalence principle, he concluded that it is actually the law of inertia whose origin is entropic. One of the people who has written extensively on the subject is Thanu Padmanabhan, where he describes all the general characteristics of a program to understand gravity as an emerging, long wavelength phenomenon (such as elasticity) and discusses a concrete framework for achieving this paradigm in the context of several recent results [13–16].

In the same logic, we notice from hydrodynamics some resemblance between the Einstein's equations and the incompressible Navier Stokes' equation which are probably the most famous and well studied nonlinear differential equations in mathematical physics. A lot of interesting phenomena such as turbulence, black holes, Big Bang, etc. are connected to these equations. Therefore, the idea to relate these systems is very promising and was proposed back in 70's in the context of the membrane paradigm [3,4]. The fluid/gravity relation reappeared in the string theory context as a particular regime of the AdS/CFT correspondence. The idea of a fluid is regularly invoked to explain the behaviour of black holes in simple terms. Suppose that you are taken in a river flowing at velocity v. If v is larger than the speed at which you can swim, then there is no chance to escape from the current. In this analogy, the black hole is the river, and the maximum swimming velocity is the speed of light. The same analogy is also invoked to explain the frame-dragging phenomenon around a Kerr black hole. From all those remarks and observations, why not envisaging a model in which the origin of gravity is a fluid?

This is the main objective of this work. We must understand to which extent we can give a physical meaning to these analogies relating to hydrodynamics by proposing some models of gravitation which are the absorption model and the relativistic model. We expect to find to some extent the Laws of Gravity as known.

In order to achieve this goal, we organised the work as follows.

In the first part we portray the absorption model. We look at how gravity is generated and how does it interact with matter? We also present some limits and discussion in relation to what we know about gravity form experiment.

In the second part, namely the relativistic model, we will look at a relativistic approach to the analogies. We consider the Einstein-æther theory, where we are working in the Minkowski space-time endowed of the metric tensor field η_{ab}. The theory involves a dynamical timelike vector field v_a. the vector defines a congruence of timelike curves filling all of spacetime, like an omnipresent fluid, and so has been dubbed "æther". Firstly we describe how matter feels the æther flow, secondly how the æther flow is generated and finally some discussion about the physical senses and the limit of the model.

2. Absorption Model

In his 1687 theory, Isaac Newton postulates that space is an infinite and unalterable physical structure that exists before, within and around all objects while their states and relationships unfold at a constant rate everywhere, so space and time are absolute. By inferring that all objects with a mass approach at a constant velocity, but collide by impact proportional to their masses, Newton deduced that the material has an attractive force. His law of universal gravitation, of which he has mathematically stated, covers the entire Universe instantly (despite absolute time), or it is not really a force to be an instant interaction between all objects (despite absolute space). As conventionally interpreted, Newton's theory of motion modelled a central force without a communicating medium. Thus, Newton's theory violated the first principle of mechanical philosophy, as Descartes stated, No action at a distance. Conversely, during the 1820s when explaining magnetism, Michael Faraday inferred a field filling space and transmitting that force. Faraday conjectured that ultimately, all forces unified into one.

In 1873, James Clerk Maxwell unified electricity and magnetism as effects of an electromagnetic field whose third consequence is light, travelling at a constant speed in a vacuum. The theory of the electromagnetic field contradicted the predictions of Newton's theory of motion, unless the physical states of a luminous æther (supposed to fill all space either in matter or in a vacuum and manifest the electromagnetic field) align any phenomenon and thereby held valid the Newtonian principle relativity or invariance.

Let us take again the idea of æther but in different way. Here we still assume that the Universe is filled with an æther fluid and fix some law of motion of this latter. From this law, we will study how matter generates the æther flow (which we called the active aspect) and also how matter experiences the æther flow (this is the passive aspect), especially in the case of particle. By discussing on the Two-body problem, we will recover Newton's laws of gravitation, and find the limits of our model.

2.1 An Æther Reloaded

Consider the following model for Newtonian gravity. Space is filled with an æther fluid with a density $\rho_{æ}$, that we will assume to be fixed. We will model gravity as the interaction between matter and this æther. Specifically, we assume that matter is able to absorb æther proportionally to its own mass. The master equation describing such a phenomenon is

$$\vec{\nabla} \cdot \vec{J}_{æ} = -\frac{\rho}{\tau} \ , \tag{2.1.1}$$

where $\vec{J}_{æ} = \rho_{æ}\vec{v}_{æ}$ is the æther mass current (or momentum flux density), $\vec{v}_{æ}$ is its velocity field, ρ is the normal matter mass density, and τ is a time constant. This constant quantifies how efficient mass is able to absorb æther.

Equation (2.1.1) must be understood as a continuity equation. So let us take a closed domain Ω containing a massive body and look at the flow of the æther fluid through the domain (see Figure 2.1). The mass of the æther in the domain Ω varies proportionally to the mass of matter contained in Ω and the total mass in Ω is conserved, thus we can write

$$\int_{\Omega} \frac{\partial \rho_{æ}}{\partial t} \mathrm{d}\vec{x} + \int_{\partial\Omega} \rho_{æ}\vec{v}_{æ} \cdot \mathrm{d}\vec{S} = -\int_{\Omega} \frac{\partial \rho}{\partial t} \mathrm{d}\vec{x} \ . \tag{2.1.2}$$

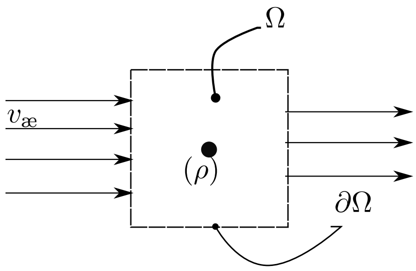


Figure 2.1: Flow of æther fluid through the domain Ω by looking a particular direction.

By using the divergence theorem, we obtain locally

$$\frac{\partial \rho_{æ}}{\partial t} + \vec{\nabla} \cdot \vec{J}_{æ} = -\frac{\partial \rho}{\partial t} \ . \tag{2.1.3}$$

We assumed that $\rho_{æ}$ is fixed so its variation is null and the right-hand side is the quantity of æther absorbed by the body which is proportional to its own mass. Thus we obtain Equation (2.1.1).

Let us consider a particle with mass m, a bounded domain Ω which contains the particle and its boundary $\partial\Omega$. The amount of æther mass absorbed by the particle per time is equal to

$$\int_{\Omega} \nabla \cdot \vec{J}_{æ}(t, \vec{x}) \mathrm{d}\vec{x} = -\frac{m(t)}{\tau} \ . \tag{2.1.4}$$

A particular consequence of (2.1.1) is that particles gain mass with time. So the variation of the particle's mass per unit time reads

$$\frac{\mathrm{d}m(t)}{\mathrm{d}t} = \frac{m(t)}{\tau} \ . \tag{2.1.5}$$

Note that in our model the mass of any body varies, which is not the case in our reality (not all masses vary as long they experience a gravitational force). But we can make this variation negligible by setting the magnitude of τ very large in front of the magnitude of the entire mass of the Universe itself. So we will have whatever the object in the Universe $\mathrm{d}m/\mathrm{d}t \sim 0$

2.2 Relation with Gravity

From the description of the æther model above, we will look at how matter generates an æther flow, which is the active aspect of the gravitation. Further, we will study how matter is dragged by the flow, which is the passive aspect of gravitation. This aspect tells us how matter experiences gravity.

2.2.1 Active Aspect. Now we will look at the characteristic of the flow created by matter, in particular a particle in order to make the analogies between the gravitational force of Newton and the flow created by a single particle.

Consider a punctual particle with mass m. Let us take the frame of the particle itself and put it at the centre of the coordinate system. In absence of any other matter in the space, the system is spherically symmetric. Thus the velocity $\vec{v}_{æ}$ of an æther particle situated at the distance r of matter verifies

$$\rho_{æ} \vec{\nabla} \cdot \vec{v}_{æ} = -\frac{\rho}{\tau}. \tag{2.2.1}$$

$\vec{v}_{æ}$ is radial, and depends only on r. This means that $\vec{v}_{æ} = v_{æ}(r)\vec{e}_r$, where $\vec{e}_r$ is the radial unitary vector. Hence we have

$$\vec{v}_{æ}(r) = -\frac{1}{4\pi\rho_{æ}\tau}\frac{m(t)}{r^2}\vec{e}_r \ . \tag{2.2.2}$$

This equation is obtained by the integration over a sphere which contains matter (particle) and the radius r and using the fact that $\rho(t) = m(t)\delta(\vec{x})$.

Recall that $\rho_{æ}$ and τ are two constants characterising æther. In order to properly identify them with the gravitational constants, we need to investigate the passive aspect.

2.2.2 Passive Aspect. Now we are looking at how physics undergoes the effect of an external gravity field. That is, in our model, how particles are dragged by the flow.

Let us suppose that one puts a particle with mass at the centre of the coordinate system inside a large-scale æther flow with velocity $\vec{V}_{æ}$ with respect to the rest frame of particle. The presence of matter will perturb the flow because of the absorption of æther. This absorption will create a flow of æther with velocity $\vec{v}$, so a particle of æther will have a velocity $\vec{v}_{æ}$ given by

$$\vec{v}_{æ} = \vec{V}_{æ} + \vec{v} \ . \tag{2.2.3}$$

For the case of a particle, $\vec{v}$ is given by Equation (2.2.2)

In order to describe how matter experiences gravitation (the flow), let us look at the amount $\mathrm{d}\vec{p}_{æ}$ of æther momentum which is absorbed by matter during an infinitesimal amount of time $\mathrm{d}t$. In the following, we will consider the case of a particle for simplicity.

To do so, let us consider a sphere (S) of radius R and evaluate the amount of æther momentum which enters in the sphere. We will take R very small to just have the amount of æther momentum absorbed by the particle because all æther particles which enter in the sphere will not be all absorbed by the particle.

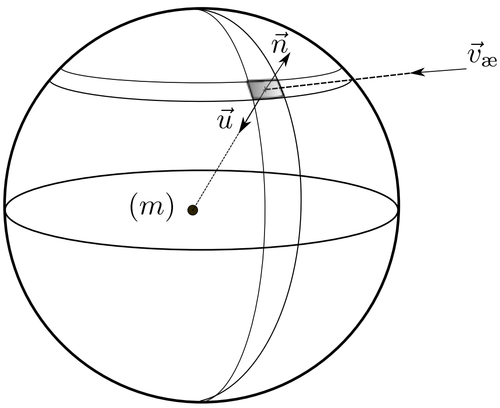


Figure 2.2: Particle of mass m absorbing an æther particle through the sphere (S)

The amount of the æther momentum which enters in the sphere (S) during the time $\mathrm{d}t$ reads

$$\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = -\int_S \rho_{æ}\vec{v}_{æ}\left(\vec{v}_{æ}\cdot\vec{n}\right)\mathrm{d}S \ , \tag{2.2.4}$$

where $\vec{v}_{æ}$ is given by the Equation (2.2.3).

In the case where the velocity $\vec{V}_{æ}$ is constant, it does not depend on the coordinate system. We are free to choose the direction of this latter as a coordinate axis because of the symmetry of the system.

Hence we obtain

$$\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = \frac{4}{3}\frac{m}{\tau}\vec{V}_{æ} \; . \tag{2.2.5}$$

This quantity will increase the momentum $\vec{p}$ of the particle in such a way that the force undergone by the particle is given by

$$\vec{F} = \frac{4}{3}\frac{m}{\tau}\vec{V}_{æ} \; . \tag{2.2.6}$$

In other to further understand this result, let us consider the case of two particles in space and look at the force that each one exerts on the other.

The case of two particles. Two particles with masses m_1 and m_2 at rest and respectively localized at the points O_1 and O_2, absorb æther. Thus the particles (1) and (2) respectively create an æther flow each one with velocity $\vec{v}_1$ and $\vec{v}_2$.

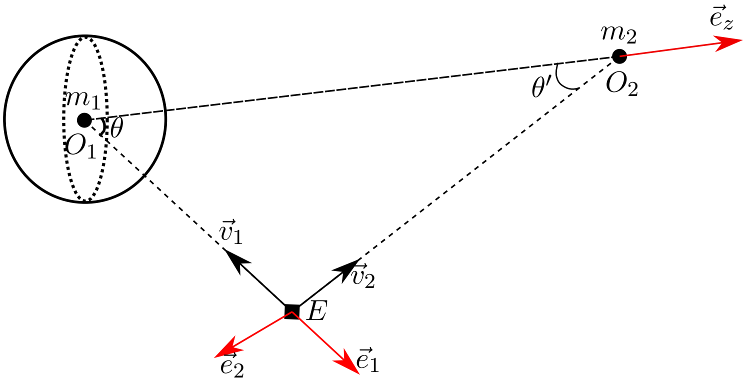


Figure 2.3: Two particles at rest undergo the force generated the flow of æther

The velocity of æther with respect to the particle (1) localised at the point E is

$$\vec{v}_{æ} = \vec{v}_1(æ/O_1) + \vec{v}_2(æ/O_1) \; , \tag{2.2.7}$$

because of the linearity of Equation (2.1.1). Here $\vec{v}_i(æ/O_j)$ is the velocity generated by the particle (i) $i = \{1, 2\}$ with respect to the frame of the particle (j). Let us denote $\vec{v}_i \equiv \vec{v}_i(æ/O_i)$ defined in the Equation (2.2.2). Since the particles are at rest, that means $\vec{v}_2(O_2/O_1) = \vec{0}$, thus we have

$$\vec{v}_{æ} = \vec{v}_1 + \vec{v}_2 \; . \tag{2.2.8}$$

We can compute by the same way as the Equation (2.2.4), the amount of æther momentum absorbed by the particle (of mass m_1 for instance). So we have

$$\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = -\lim_{r_1\to 0}\int_S \rho_{æ}\left(\vec{v}_1 + \vec{v}_2\right)\left[(\vec{v}_1 + \vec{v}_2)\cdot\vec{e}_1\right]\mathrm{d}S \; . \tag{2.2.9}$$

Let us set

$$\vec{e}_1 = \frac{\overrightarrow{O_1E}}{O_1E}; \qquad \vec{e}_2 = \frac{\overrightarrow{O_2E}}{O_2E}; \qquad \vec{e}_z = \frac{\overrightarrow{O_1O_2}}{O_1O_2}; \qquad O_1E = r_1; \qquad O_2E = r_2; \qquad O_1O_2 = r \; . \tag{2.2.10}$$

We call θ the angle between $\vec{e}_z$ and $\vec{e}_1$ (see Figure (2.3). Thus we obtain

$$(\vec{v}_1 + \vec{v}_2) \cdot \vec{e}_1 = v_1(r_1) + v_2(r_2)\frac{r_1 - r\cos\theta}{\sqrt{r_1^2 + r^2 - 2rr_1\cos\theta}} \ . \tag{2.2.11}$$

With the settings in the Equation (2.2.10), we performed the integral (2.2.9) by using the fact that the integrand is a continuous smooth function. Therefore, we can interchange the limit and the integral which make the calculation easier. Hence we finally get

$$-\frac{1}{\rho_{æ}}\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = -\frac{4}{3}\frac{4\pi}{(4\pi\rho_{æ})^2}\frac{m_1 m_2}{\tau^2}\frac{1}{r^2}\vec{e}_z \ . \tag{2.2.12}$$

So the amount of æther momentum which is absorbed per unit time by the particle of mass m_1 is

$$\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = \frac{4}{3}\frac{1}{4\pi\rho_{æ}}\frac{m_1 m_2}{\tau^2}\frac{1}{r^2}\vec{e}_z = \frac{4}{3}\frac{m_1}{\tau}\vec{v}_2(r). \tag{2.2.13}$$

Hence the force exerted by the particle (2) on the the particle (1) is given by

$$\vec{F}_{2/1} = \frac{4}{3}\frac{1}{4\pi\rho_{æ}}\frac{m_1 m_2}{\tau^2}\frac{1}{r^2}\vec{e}_z = \frac{4}{3}\frac{m_1}{\tau}\vec{v}_2(r) \ . \tag{2.2.14}$$

By doing the same computation for the particle of mass m_2 we get

$$\vec{F}_{1/2} = -\vec{F}_{2/1} \ . \tag{2.2.15}$$

Hence, according to Newtons law of gravitation, we can make an analogy with our model. Before that, let us recall the Newton law of gravitation. The attractive force between two mass as in the case described by the figure 2.3 is given by

$$\vec{F}_{1/2}^{\ \mathrm{Newton}} = -\vec{F}_{2/1}^{\ \mathrm{Newton}} = -\frac{G m_1 m_2}{r^2}\vec{e}_z \ , \tag{2.2.16}$$

where $G \simeq 6.67408 \times 10^{-11}\ \mathrm{kg}^{-1}.\mathrm{m}^3.\mathrm{s}^{-2}$ is the gravitational constant and we can set our analogy by

$$G = \frac{4}{3}\frac{1}{4\pi\rho_{æ}\tau^2} \ . \tag{2.2.17}$$

From those results, we can understand how two matter particles interact through the æther flow and how this interaction can be interpreted as a gravity force of Newton. This conclusion would not be easy to make from Equation (2.2.6) which tells us how matter experiences the æther flow that we assumed constant. Since we are in the case of a particle, the force found in the Equation (2.2.14) is similar to the force in (2.2.6), simply by replacing the constant velocity $\vec{V}_{æ}$ by the velocity of æther created by the particles (2).

Indeed, in the case of the particle, we took the limit $r_1 \to 0$. Now suppose that the particle is spherical and has a radius of R, we take the limit $r_1 \to R$. By doing the calculation from the beginning, we obtain

$$-\frac{1}{\rho_{æ}}\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = \frac{2\pi}{(4\pi\rho_{æ})^2}\frac{m_1 m_2}{\tau^2}\left(\frac{2\,r^4 + 2\,r^3 R - rR^3 - R^4}{3\sqrt{r^2 + 2\,rR + R^2}r^2R^3} - \frac{2\,r^4 - 2\,r^3 R + rR^3 - R^4}{3\sqrt{r^2 - 2\,rR + R^2}r^2R^3} - \frac{2}{r^2}\right)\vec{e}_z \ . \tag{2.2.18}$$

By setting $R = xr$, where $0 < x < 1$ is a proportionality coefficient, we have the force undergone by the spherical particle which is written as follows

$$\vec{F}'_{2/1} = -\frac{3}{8}\left(-\frac{x^4 + x^3 - 2\,x - 2}{3\,\sqrt{x^2 + 2\,x + 1}x^3} + \frac{x^4 - x^3 + 2\,x - 2}{3\,\sqrt{x^2 - 2\,x + 1}x^3} - 2\right)\vec{F}_{2/1}\ . \tag{2.2.19}$$

After few developments and simplifications, for $0 < x < 1$, We obtain

$$\vec{F}'_{2/1} = \vec{F}_{2/1}\ . \tag{2.2.20}$$

In the case of spherical symmetry, equation (2.2.20) shows that the force undergone by the particle does not depend on its radius.

Now let us discuss all the results that we have got so far, from the active to the passive aspect. In order to see first how the model can be useful and also its limits.

2.3 Discussion

2.3.1 Some Analogies. Now we know how the æther flow is generated, how matter experiences this flow and the standard formulation of Newton's gravity formulation. We can start to make some interpretation from our model to the Newton gravitation as we did with the Equation (2.2.17).

Let us start by highlighting that the interpretation of the gravitational field which is defined in Newtonian gravity as follows (in the case of two particles)

$$\vec{g}_{1/2}^{\,\mathrm{Newton}} = \frac{1}{m_2}\vec{F}_{1/2}^{\,\mathrm{Newton}} = -\frac{Gm_1}{r^3}\vec{r}\ . \tag{2.3.1}$$

This equation represents the gravitational field created by the particle (1) at the position of the particle (2). The interpretation of the latter in our model for the case of two particles is given as

$$\vec{g}_{1/2} = -\frac{4}{3}\frac{1}{4\pi\rho_{æ}\tau^2}\frac{m_1}{r^3}\vec{r} = \frac{4}{3}\frac{\vec{v}_1(r)}{\tau}\ , \tag{2.3.2}$$

where $\vec{v}_1(r)$ is the velocity of æther localized at the distance r from the particle (1). We can better understand this by taking the scenario that a particle (2) absorbed an æther particle who follows the æther flow generated by the particle (1). Hence, the particle (2) will also follow the æther flow as well, that means it will experience a force of gravity described at Equation (2.2.14).

In the case where the matter is not a point particle, there is no guarantee that we can write the force that a body undergoes from the æther flow as

$$\vec{f} = m_G\vec{g} = -m_G\vec{\nabla}\Phi\ , \tag{2.3.3}$$

where m_G is the gravitational mass and Φ the gravitational potential. In this case, the force will depend of the body shape.

We want to have at least an order of magnitude of the parameters τ and $\rho_{æ}$ according to the analogies made. As mentioned above τ should be very large to keep the variation of the mass during the absorption of the æther almost null. We chose an order of magnitude of τ equal to the age of the Universe, just to show how to turn off the parameters in order to have a physical sense. We obtain

$$\tau \sim 14\times 10^{12}\ \text{years} \qquad \rho_{æ} = \frac{4}{3}\frac{1}{4\pi G\tau^2} \sim 10^{-30}\ \mathrm{kg.m^{-3}}\ . \tag{2.3.4}$$

We find a density which is 1000 times smaller than the Universe density. The Universe density is in the order [1] of $10^{-27}\mathrm{kg.m}^{-3}$. That means $\rho_{\text{universe}} = 10^3\rho_{æ}$. Indeed this comparison is physically meaningless, because here æther is different from matter. We did this comparison just to have an order of magnitude in spirit.

2.3.2 The Force Exerted On a Particle in Motion By The Æther. Let us take again the case of the two particles and this time let us assume that they move like in Two-body problem (from celestial mechanic). We will see that the force exerted on each particle depends on its velocity, which is not the case in Newtonian mechanics. Further on, we will study the movement of two bodies, for instance the Earth and the Sun, and will see how long the system survives in our model.

Previously we supposed that the particles were not moving. In that case, an æther particle has a velocity $\vec{v}_{æ} = \vec{v}_1 + \vec{v}_2$ with respect to the rest frame of both particles. Now the particles are in motion and at the same time, they create a flow of æther. To determine the force undergone by the particle (1), we must calculate the velocity of æther with respect to this particle.

$$\vec{v}_{æ} = \vec{v}_1(æ/O_1) + \vec{v}_2(æ/O_1) = \vec{v}_1 + \vec{v}_2 + \dot{\vec{r}}\,, \tag{2.3.5}$$

where we used $\dot{\vec{r}} = \dfrac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \dfrac{\mathrm{d}\overrightarrow{O_1O_2}}{\mathrm{d}t} = \vec{v}_2(O_2/O_1)$.

Hence by applying the same logic as the previous case to compute the amount of æther momentum absorbed, we can write

$$-\frac{1}{\rho_{æ}}\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = \lim_{r_1\to 0}\int_S \left(\vec{v}_1 + \vec{v}_2 + \dot{\vec{r}}\right)\left(\left(\vec{v}_1 + \vec{v}_2 + \dot{\vec{r}}\right)\cdot\vec{e}_1\right)\vec{e}_1\mathrm{d}S\,. \tag{2.3.6}$$

Using the same setting as in the previous case (see Equation (2.2.10), and Figure 2.3) and applying the same logic, we get

$$-\frac{1}{\rho_{æ}}\frac{\mathrm{d}\vec{p}_{æ}}{\mathrm{d}t} = -\frac{4}{3}\frac{m_1}{\rho_{æ}\tau}\vec{v}_2(r) - \frac{4}{3}\frac{m_1}{\rho_{æ}\tau}\dot{\vec{r}}\,. \tag{2.3.7}$$

Hence, we find the force experienced by particle (1) as

$$\vec{F}_1 = \frac{4}{3}\frac{m_1m_2}{4\pi\rho_{æ}\tau^2}\frac{\vec{r}}{r^3} + \frac{4}{3}\frac{m_1}{\tau}\dot{\vec{r}}\,. \tag{2.3.8}$$

Similarly, the force that is experienced by particle (2) is written

$$\vec{F}_2 = -\frac{4}{3}\frac{m_1m_2}{4\pi\rho_{æ}\tau^2}\frac{\vec{r}}{r^3} - \frac{4}{3}\frac{m_2}{\tau}\dot{\vec{r}}\,. \tag{2.3.9}$$

We notice that

$$\vec{F}_1 \neq -\vec{F}_2\,. \tag{2.3.10}$$

Hence, the force of gravity of this model depends on the velocity of matter. This is not the case in the Newtonian mechanics, where the gravitational force does not depend on the motion of matter.

This force (2.3.8 or 2.3.9) contains two effects: the effect of Newtonian gravity and the effect of the friction force of the particle with the ambient æther. In addition to the gravitational force that we are trying to model, there is a friction force that has no counterpart in Newtonian gravitation. The latter means that the force exerted on the particle depends on its relative velocity with respect to the other particle. This is particularly annoying, in the way that the model will not be able to explain for instance, the system Sun-Earth as we know. It will rather predict that the Earth experiences a friction force during its motion, which will reduce its radius of orbit until it hits the Sun.

[1] from the WMAP'S Universe website : https://wmap.gsfc.nasa.gov/universe/uni_matter.html

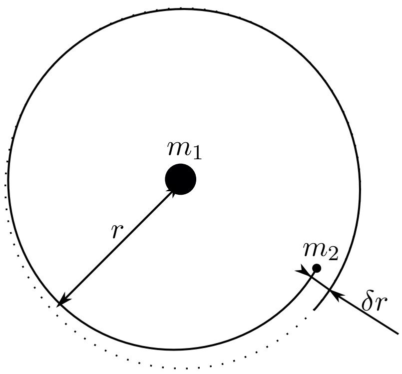


Figure 2.4: Effect of the friction force on the orbit: from a circular orbit, we observe a reduction of the radius.

2.3.3 Life-time of the Solar System. We consider the previous system as the Sun-Earth system where the particle (1) is the Sun and the particle (2) is the Earth. We assume for simplicity that the Sun is at rest, in other words $m_2 \ll m_1$.

Remembering that the mass here is not constant, we can rewrite the equation of motion as

$$\ddot{\vec{r}} + \frac{1}{\tau}\dot{\vec{r}} = -Gm_1\frac{\vec{r}}{r^3} - \frac{4}{3}\frac{1}{\tau}\dot{\vec{r}} \ . \tag{2.3.11}$$

Multiplying the equation by $\dot{\vec{r}}$ we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\dot{\vec{r}}^2 - \frac{Gm_1}{r}\right) = -\frac{7}{3\tau}\dot{\vec{r}}^2 \ , \tag{2.3.12}$$

where the right-hand side represents the variation of the energy of the system. As we can see, the energy decreases.

Just to have an idea of the order of magnitude of the life-time of the Sun-Earth system, we assume that the energy loss is very slow (because τ is very large). We can assume that the trajectory of the Earth is circular, with slowly changing velocity and radius.

For a circular trajectory, the velocity and radius are related by

$$v^2 = \dot{\vec{r}}^2 = \frac{Gm_1}{r} \ . \tag{2.3.13}$$

Using this relation, we can rewrite the Equation (2.3.12) as follow

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{r}\right) = \frac{14}{3\tau}\frac{1}{r} \ . \tag{2.3.14}$$

It leads us to

$$r(t) = r_0 \exp\left(-\frac{14t}{3\tau}\right) \ . \tag{2.3.15}$$

By assuming that the initial radius of the Sun-Earth is $r_0 = 1\text{u.a.} = 150 \cdot 10^6\text{km}$ and the radius of the Sun is $R_1 = 695,510\text{km}$. Since the Sun is larger and heavier than the Earth, the latter (the Earth) can be considered as a point particle of mass m_2. So from this approximation, the Earth would hit the Sun (i.e. $r(t) = R_1$) after a time:

$$t_{\text{collision}} = \frac{3}{14}\frac{R_1}{r_0}\tau \sim 1.15\tau \ . \tag{2.3.16}$$

Hence the Solar system would collapse after a time of order τ. However, the issue can be circumvented since τ is a free parameter, which can be large as desired. We can thus, reproduce the Newtonian gravitation up to any precision.

Let us take the velocity of æther from (2.2.2) and the setting of the characteristic constants of æther in (2.3.4). We can see that for r, the distance to the source of æther flow, small and large mass m, the velocity will be larger than the speed of the light. That we know from general relativity is the limit speed.

This suggests us to think about another model, which will take into account the relativistic constraint. In fact this model is all about Newtonian mechanics.

2.3.4 Production of Æther. One of the questions that can be asked in the face of such a model is whether the æther is infinite. Indeed, the absorption of æther will cause the latter to decrease inside space. So the question is, can we adopt the model so that the æther is provided continuously?

Let's take the model from the beginning and rewrite the absorption law (2.1.1) by introducing a production term for æther, we get

$$\vec{\nabla} \cdot \vec{J}_{æ} = -\frac{\rho}{\tau} + \sigma \ , \tag{2.3.17}$$

where σ is a constant that represents the rate of production of æther per time unit. σ behaves exactly like the cosmological constant. From the reference [7], the cosmological constant is not a strictly relativistic concept: in Newtonian physics, it can be added to the Poisson's equation as $\Delta\Phi + \Lambda = 4\pi G\rho$, where Φ is the gravitational potential, ρ is the density of matter and Λ is the cosmological constant.

An important feature of the absorption model, as described here, is to provide a description of gravity where the active and passive aspects are explained simultaneously by the Equation (2.1.1). It is different in Newton gravitation, where the passive aspect is described by Newton's second law $\vec{F} = m\vec{a}$ and the active aspect by the Poisson's equation $\Delta\Phi = 4\pi G\rho$.

3. Thermal Model

To introduce this new chapter, it would be wise to go through the theory of special relativity that will form the background of thermal model.

Special relativity (SR, also known as special theory of relativity or STR) is the generally accepted and experimentally well-confirmed physical theory of the relationship between space and time. In Albert Einstein's original formulation, it is based on two postulates:

1. The laws of physics are invariant in all inertial systems (i.e. non-accelerating reference frames): **the Lorentz invariance**.
2. The speed of light in vacuum is the same for all observers, regardless of the movement of the light source.

Special relativity was originally proposed by Albert Einstein in an article published on the 26th September 1905 entitled "On the Electrodynamics of Moving Bodies". The incoherence of Newtonian mechanics with Maxwell's equations of electromagnetism and the absence of experimental confirmation of a hypothetical luminous æther have led to the development of special relativity, which corrects mechanics to deal with situations involving all movements. Today, special relativity is the most accurate model of motion at any speed when gravitational effects are negligible. Despite this, the Newtonian mechanical model is still valid as a simple and very accurate approximation at low speed compared to the speed of light.

It was only when Einstein developed general relativity, introducing a curved space-time to incorporate gravity, that the expression "special relativity" was used.

The theory is "special" in the sense that it applies only to the particular case where space-time is flat, i.e. when the curvature of space-time described by the Riemann tensor which is linked to the energy-momentum tensor and involving gravity is negligible. In order to properly incorporate gravity, Einstein formulated general relativity (GR) in 1915.

In this chapter, in contrast to Einstein's description of gravity, we will remain in special relativity and assume the existence of a fluid that fills all space-time. This fluid will be coupled to matter with the effect of imposing a privileged direction of time. This framework has already been presented in Einstein-æther's theory [6, 10, 11].

In GR, the space-time structure is determined by a dynamic metric tensor field g_{ab} and nothing else, and the theory is both diffeomorphism invariant and locally Lorentz invariant . Einstein-æther theory is the extension of GR that incorporates a dynamical unit timelike vector field v^a the "æther" that breaks down the local Lorentz symmetry into a 3D rotation subgroup. Direct coupling of matter to æther would violate local Lorentz symmetry while preserving the invariance of diffeomorphism [11].

3.1 A Relativistic Æther

Could there be an æther after all and we have just not yet observed it? By æther we do not necessary mean to mechanical medium whose deformations correspond to electromagnetic fields, but rather a locally preferred state of rest at each point of spacetime, determined by some unknown physics up to now. Considerations of quantum gravity have in multiple ways led to this question.

This model is built on the basis of the Einstein-æther theory (or shortly æ-theory). Spacetime in our case is considered flat so it is endowed with a Minkowski metric $\eta_{ab} = \mathrm{diag}(-1, +1, +1, +1)$. To create gravitation without the principle of curvature of spacetime, we fill spacetime with a congruence timelike curves, like an omnipresent æther fluid. The motion of the æther fluid depends on the coupling with matter in spacetime. We denote by v^a the four-velocity of æther which is a dynamical unit timelike vector field as described in the æ-theory by the references [1, 2, 6, 8, 10]. Unit timelike vector field means $v^a v_a = -1$.

The interaction between æther and matter is described by the dynamical preferred frame contained in the æ-theory. So, for matter in motion with four-velocity u^a, we define a parameter

$$\gamma = -v_a u^a \ , \tag{3.1.1}$$

which is the Lorentz factor of the particle in the æther's frame (or vice versa). This parameter arises as the coupling of æther with matter.

In order to see how this coupling can be interpreted as gravity, let us look at how a particle moves under the influence of a field of æther v^a.

3.2 Passive Aspect of Gravity

Consider a body with rest mass m. The body will be treated as a point particle. The action in this case can be deduced from the standard one-particle action ([7], page 42). In the same spirit as the references [1, 8], we assume that the action of the massive particle interacting with æther reads

$$S = -m \int \mathrm{d}\tau F(\gamma) \ , \tag{3.2.1}$$

where the integral is along the particle's worldline parametrized by τ, which is the particle's proper time. let u^a be the unit four-velocity of the particle, and v^a is æther velocity (or shortly æther).

In the case where the velocity of the particle is close to the æther velocity we have $\gamma \sim 1$. We can expand F as follow

$$F(\gamma) = 1 - \alpha(\gamma - 1) - \alpha'(\gamma - 1)^2 + \cdots = 1 + \alpha(v_a u^a + 1) + \frac{\alpha'}{2}(v_a u^a + 1)^2 + \cdots \ , \tag{3.2.2}$$

where α and α' are constants. The quantity $v_a u^a$ is of order u^2, the square of the velocity of the particle in the æther frame. We will see that the terms of order u^2 constitute the Newtonian limit of this theory.

In order to recover the behaviour of gravity, let us look for the equation of motion of the body.

3.2.1 Equation of Motion. The equation of motion of the body whose action is given by (3.2.1), is obtained by relying on the trick which consists of artificially introducing an arbitrary parameter λ along the world-line of the particle, namely;

$$S[x^a] = -m \int F(\gamma)\sqrt{-\eta_{ab}\dot{x}^a \dot{x}^b}\mathrm{d}\lambda \ , \tag{3.2.3}$$

where we denoted $\dot{x}^a \equiv \mathrm{d}x^a/\mathrm{d}\lambda$. By setting

$$L(x^a, \dot{x}^a) = F(\gamma)\sqrt{-\eta_{ab}\dot{x}^a \dot{x}^b} \ , \tag{3.2.4}$$

one shows that the functional derivative of S reads

$$\frac{\delta S}{\delta x^a} = \frac{\partial L}{\partial x^a} - \frac{\mathrm{d}}{\mathrm{d}\lambda}\frac{\partial L}{\partial \dot{x}^a} \ . \tag{3.2.5}$$

From the Hamilton's least action principle, the equation of motion of a particle localised by x^a reads

$$\frac{\delta S}{\delta x^a} = 0 \implies \left(F(\gamma) - \gamma F'(\gamma)\right)\frac{\mathrm{d}u_a}{\mathrm{d}\tau} - \omega_{ab}u^b F'(\gamma) + \frac{\mathrm{d}\gamma}{\mathrm{d}\tau}\left(v_a - \gamma u_a\right)F''(\gamma) = 0 \ , \tag{3.2.6}$$

where we denote $\omega_{ab} = (\mathrm{curl}\mathbf{v})_{ab} = \partial_a v_b - \partial_b v_a = v_{b,a} - v_{a,b}$. After some simplification, the equation can be rewritten as follow

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left[(F - \gamma F')u^a\right] = F'\omega^a{}_b u^b - \frac{\mathrm{d}\gamma}{\mathrm{d}\tau}F''v^a \ . \tag{3.2.7}$$

To give an explanation of this equation of motion of a body is not easy under this form. Let us consider a Newtonian limit of the equation. We assume that there is a frame such that both the particle and æther have a small velocity. In that frame, we can write

$$(u^\mu) = \gamma_u(1, \vec{u}) \qquad \text{with} \quad \gamma_u \equiv (1 - \vec{u}\cdot\vec{u})^{-1/2} \ ; \tag{3.2.8}$$

$$(v^\mu) = \gamma_v(1, \vec{v}) \qquad \text{with} \quad \gamma_v \equiv (1 - \vec{v}\cdot\vec{v})^{-1/2} \ . \tag{3.2.9}$$

We consider the linear part of the expansion of F given in Equation (3.2.2)

$$F(\gamma) = 1 - \alpha(\gamma - 1) \ . \tag{3.2.10}$$

The equation of motion can be rewrite as

$$(1+\alpha)\frac{\mathrm{d}u^a}{\mathrm{d}\tau} = -\alpha\left(\partial^a v_b - \partial_b v^a\right)u^b \ . \tag{3.2.11}$$

Splitting this equation into spatial and temporal components we get

$$\text{for } a = 0 \qquad (1+\alpha)\frac{\mathrm{d}u^0}{\mathrm{d}\tau} = -\alpha\left(\partial^0 v_i - \partial_i v^0\right)u^i \ ; \tag{3.2.12}$$

$$\text{for } a = j \qquad (1+\alpha)\frac{\mathrm{d}u^j}{\mathrm{d}\tau} = -\alpha\left(\partial^j v_0 - \partial_0 v^j\right)u^0 - \alpha\left(\partial^j v_i - \partial_i v^j\right)u^i \ . \tag{3.2.13}$$

The easiest form of this equation is obtained if we assume that æther is irrotational and stationary. That means $\partial_j v^i - \partial_i v^j = 0$ and $\partial_0 v^i = 0$. In that case, we have

$$\text{for } a = 0 \qquad (1+\alpha)\frac{\mathrm{d}\gamma_u}{\mathrm{d}\tau} = \alpha u^i \partial_i \gamma_v \ ; \tag{3.2.14}$$

$$\text{for } a = j \qquad (1+\alpha)\frac{\mathrm{d}u^j}{\mathrm{d}\tau} = \alpha\gamma_u\partial^j\gamma_v \ . \tag{3.2.15}$$

Putting together Equations (3.2.15) and (3.2.14) and using $\gamma_u \mathrm{d}\tau = \mathrm{d}t$, we get

$$(1+\alpha)\gamma_u\frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = \alpha\left(\vec{u}\cdot\vec{\nabla}\gamma_v\right)\vec{u} + \alpha\vec{\nabla}\gamma_v \ . \tag{3.2.16}$$

3.2.2 Newtonian Limit. In the Newtonian limit, we assume that the motion of the particle and the æther flow are slow (that means non-relativistic æther), allow to make an approximation. Let us take $u \ll 1$ and $v \ll 1$, and work at lowest non-vanishing order in u and v. Hence we have $\gamma_v \simeq 1 + v^2/2$; $\gamma_u \simeq 1 + u^2/2$ and finally the equation of motion reads

$$\frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = \frac{\alpha}{2(1+\alpha)}\vec{\nabla}v^2 \ . \tag{3.2.17}$$

So the Lorentz factor of the æther flow would be analogous to a gravitational potential. In the regime of a non-relativistic æther ($\gamma_v \approx 1 + v^2/2$), we can identify the gravitational potential with $\alpha v^2/2(1+\alpha)$.

Equation (3.2.17) shows how, within the Newtonian limit of our model, the particle undergoes a force derived from a potential proportional to the square of the flow velocity of æther. In fact, we have here a law of the passive aspect of our model. This law behaves in the same way as Newton's gravitational force. The analogy appears clearer by comparing this model with Newtonian gravitation.

In Newtonian gravity, the equation of motion of a particle of inertial mass m in the gravitational field, derived from the gravitational potential Φ, is

$$\frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2} = -\vec{\nabla}\Phi \ . \tag{3.2.18}$$

We see that Equations (3.2.17) and (3.2.18) are identical if we make the identification

$$v^2 = -\frac{2(1+\alpha)}{\alpha}\Phi. \tag{3.2.19}$$

From the latter, we see that the flow of the æther creates a force that behaves within this limit as the gravitational force. Indeed, the question that arises in our minds is to know the cause of this flow. This is the active part of our model.

3.3 Active Aspect

This section aims to describe how matter creates an æther flow or in other words, how matter creates gravity. We start from the result obtained in the previous section in the case of the Newtonian limit and further, we will propose the field equation of the æther in the relativistic case.

3.3.1 Newtonian Limit. In Newtonian gravitation, the gravitational potential is determined by Poisson's equation:

$$\Delta\Phi = 4\pi G\rho \ , \tag{3.3.1}$$

where ρ is the gravitational matter density and G is Newton's gravitational constant. So we can write in the same as the Equation (3.2.19), the field equation of the æther flow.

$$\alpha \in]-\infty, -1] \cup [0, \infty[\qquad \text{and} \qquad \vec{\nabla}^2 v^2 = -\frac{8(1+\alpha)\pi G\rho}{\alpha}. \tag{3.3.2}$$

It shows how the presence of matter should create an æther flow. The constant α describes the ability of matter, as well as its density, to create this flow.

Once more, we would like to understand what is the phenomenon or phenomena behind this equation.

In order to find a phenomenon which can explain the Equation (3.3.2), we consider the fluid aspect of the æther. Assume that the æther is a perfect fluid and that its equation of motion is given by the Euler equation for a stationary flow ($\partial_t \vec{v} = \vec{0}$) which reads

$$\rho_{æ}(\vec{v} \cdot \vec{\nabla})\vec{v} = \vec{f} \ , \tag{3.3.3}$$

where $\rho_{æ}$ is the æther density and $\vec{f}$ is the sum of the body forces (forces per unit volume) acting on the æther fluid. Let us take the divergence of this equation, then

$$\rho_{æ}\vec{\nabla} \cdot (\vec{v} \cdot \vec{\nabla})\vec{v} = \vec{\nabla} \cdot \vec{f} \ , \tag{3.3.4}$$

if we assume that the æther density is constant. If we rewrite the left-hand side in terms of indices, we get

$$\vec{\nabla} \cdot (\vec{v} \cdot \vec{\nabla})\vec{v} = \partial_i(v_j \partial_j v_i) = (\partial_i v_j)(\partial_j v_i) + v_j \partial_i \partial_j v_i \ . \tag{3.3.5}$$

On the other hand, the term in which we are interested is

$$\frac{1}{2}\Delta v^2 = \frac{1}{2}\partial_i\partial_i(v_j v_j) = \partial_i(v_j \partial_i v_j) = (\partial_i v_j)(\partial_i v_j) + v_j \partial_i \partial_i v_j \ . \tag{3.3.6}$$

Equations (3.3.5) and (3.3.6) are very similar, apart from some inversions of indices i, j. However, if the flow is irrotational, i.e. $\vec{\nabla} \times \vec{v} = \vec{0}$, i.e. $\partial_i v_j = \partial_j v_i$, then

$$\frac{1}{2}\Delta v^2 = \vec{\nabla} \cdot (\vec{v} \cdot \vec{\nabla})\vec{v} \ , \tag{3.3.7}$$

and hence the divergence of Euler's equation reads

$$\rho_{æ}\frac{1}{2}\Delta v^2 = \vec{\nabla} \cdot \vec{f} \ . \tag{3.3.8}$$

In order to simulate a gravitation effect, we thus need to find a force applied on the fluid such that its divergence is proportional to the density of matter

$$\vec{\nabla} \cdot \vec{f} \propto -\rho \ . \tag{3.3.9}$$

One idea can come from thermodynamics. If we assume that the force $\vec{f}$ derives from pressure

$$\vec{f} = -\vec{\nabla} P_{æ} \ , \tag{3.3.10}$$

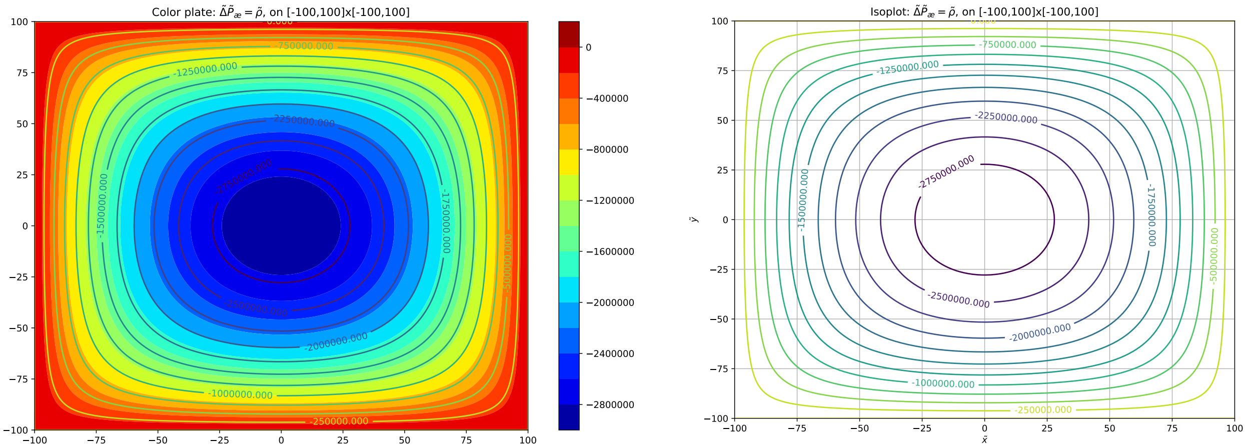
then we must have

$$\Delta P_{æ} = \frac{4(1+\alpha)\pi G}{\alpha}\rho\rho_{æ} \ . \tag{3.3.11}$$

In order to have an idea of pressure in the space, let us plot its profile. First, we transform the quantities in dimensionless quantities. Let us set $x/\tilde{x} = y/\tilde{y} = L$, the dimension of space with respect to the diameter of the Milky Way, $P_0 = 4(1+\alpha)\pi G \rho_{æ}^2 L^2/\alpha$, representing the pressure of æther that it would create by itself, $\tilde{\rho} = \rho/\rho_{æ}$, the relative density with respect to the æther's density and $\tilde{P}_{æ} = P_{æ}/P_0$, the relative pressure. Thus we have

$$\tilde{\Delta}\tilde{P}_{æ} = \tilde{\rho}. \tag{3.3.12}$$

To resolve numerically this partial differential equation (PDE), we need the boundary conditions. We suppose that at the boundaries, the pressure is equal to P_0, which means $\tilde{P}_{æ} = 1$. We assume that matter is uniformly distributed in space, so we take ρ as the density of matter in the Universe. As presented in chapter 2, $\rho = 1000\rho_{æ}$. Hence we obtain the profiles represented in Figure 3.1



(a) (b)

Figure 3.1: Profiles of the pressure in space with ρ constant

From Figure 3.1, we can see that the presence of matter depressurizes æther, thus creating a kind of diffusion phenomenon that causes the movement of æther particles from high pressure zones to low pressure zones. In a scenario of spherical symmetry, we have something that behaves like gravitation.

Now, in addition to the stationary and irrotational flow of the æther, let us assume that æther is a perfect gas. Its equation of state reads

$$P_{æ} = \kappa \rho_{æ} T_{æ} \ , \tag{3.3.13}$$

where κ is a characteristic constant of æther and $T_{æ}$ is its temperature. This assumption leads to

$$\Delta T_{æ} = \frac{4(1+\alpha)\pi G}{\alpha\kappa}\rho \ . \tag{3.3.14}$$

This is reminiscent of the heat transport equation (thermal diffusion) in a medium, which reads

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c_{\text{vol}}}\Delta T + \frac{\sigma_{\text{vol}}}{c_{\text{vol}}} \ , \tag{3.3.15}$$

where λ is the thermal conductivity of the medium, c_{vol} its volumic thermal capacity, and σ_{vol} is the volumic source of heat (power per unit volume). In the stationary regime, this becomes

$$\Delta T = -\frac{1}{\lambda}\sigma_{\text{vol}} \ . \tag{3.3.16}$$

Comparing this with $\Delta T_{æ} \propto \rho$, we conclude that, in this analogy, matter density plays the role of a negative source of heat for æther.

In short, the presence of matter in space causes a temperature change of æther. This change in temperature induces a change in pressure of æther, which in turn generates an æther flow. This flow exerts on any other body, a force behaving like gravity according to our analogy.

This result is very interesting. It paves the way towards a possible link between thermodynamics and gravity. This link has already been invoked in the references [9,12–17], emphasizing the fact that gravity can be considered as an emerging phenomenon.

Hitherto, we have only studied the Newtonian limit of our model. However, our main objective was to study the relativistic property of this model.

Although the Equation (3.2.7) describes the action of æther on matter, it remains difficult from this form to highlight the gravitational effect of this action. It would be interesting to study how the dynamics of æther is created by matter and to explore the interaction between two bodies.

3.3.2 Relativistic Level. It has been suggested in the literature, some models that describe the Universe immersed in an aether field. Among those models, the æ-theory is the most current and offer a description of the dynamic of æther field v^a by an action reads [1, 2, 6, 10]

$$S_{\text{æ}}[v^a] = \frac{-1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + K^{ab}{}_{mn}\nabla_a v^m \nabla_b v^n + \lambda\left(g_{ab}v^a v^b + 1\right)\right) \ , \tag{3.3.17}$$

where

$$K^{ab}{}_{mn} = c_1 g^{ab} g_{mn} + c_2 \delta^a_m \delta^b_n + c_3 \delta^a_n \delta^b_m + c_4 v^a v^b g_{mn} \ . \tag{3.3.18}$$

The coefficients $c_{i,\ i=1,2,3,4}$ are dimensionless constants, R is the Ricci scalar and λ is a Lagrange multiplier that enforces the unit constraint. In our model we use this description but rather of the curved space-time, we take the Minkowski metric $\eta_{\mu\nu}$. Then we can rewrite equations (3.3.17) and (3.3.18) as follow

$$S_{\text{æ}}[v^a] = \frac{-1}{16\pi G}\int \mathrm{d}^4x\left(K^{ab}{}_{mn}\partial_a v^m \partial_b v^n + \lambda\left(\eta_{ab}v^a v^b + 1\right)\right) \ , \tag{3.3.19}$$

where

$$K^{ab}{}_{mn} = c_1 \eta^{ab} \eta_{mn} + c_2 \delta^a_m \delta^b_n + c_3 \delta^a_n \delta^b_m + c_4 v^a v^b \eta_{mn} \ . \tag{3.3.20}$$

If we consider many particles of mass m_p, localised by x^a_p, with $p = \{1, 2, \cdots, N\}$, then the interaction between æther and these particles is described by the action

$$S = S_{\text{æ}} + \sum_{p=1}^{N} S_p = \frac{-1}{16\pi G}\int \mathrm{d}^4x\left(K^{ab}{}_{mn}\partial_a v^m \partial_b v^n + \lambda\left(\eta_{ab}v^a v^b + 1\right)\right) - \sum_{p=1}^{N} m_p \int \mathrm{d}\tau_p F(\gamma_p) \ . \tag{3.3.21}$$

Let us rewrite the second term in the right-hand side, we have

$$\sum_{p=1}^{N} S_p = -\sum_{p=1}^{N}\int \mathrm{d}^4x\ \frac{m_p}{u^0_p}\delta_D(\vec{x} - \vec{x}_p(t))F(\gamma_p) \ , \tag{3.3.22}$$

where $\gamma_p = -v_a u^a_p$ and δ_D is the Dirac function.

The æther field equation is given by $\delta S/\delta v^a = 0$, which is written

$$\partial_b K^{ba} = c_4(v^b \partial_b v_c)\partial^a v^c + \lambda v^a - 8\pi G \sum_{p=1}^{N} \frac{m_p}{u^0_p}\delta_D\left(\vec{x} - \vec{x}_p(t)\right) u^a_p F'(\gamma_p) \ , \tag{3.3.23}$$

where $K^b{}_a = K^{bc}{}_{ad}\partial_c v^d$.

$$\begin{aligned} \partial_b K^{ba} &= \partial_b\left(\eta^{ae}K^b{}_e\right) \ , \\ \partial_b K^{ba} &= c_1\left(\partial\cdot\partial\right)v^a + (c_2 + c_3)\partial^a\left(\partial\cdot\mathbf{v}\right) \\ &\quad + c_4\left(\left(\partial\cdot\mathbf{v}\right)\left(\mathbf{v}\cdot\partial\right)v^a + \left(\mathbf{v}\cdot\partial\right)\left(\mathbf{v}\cdot\partial\right)v^a + \mathbf{v}\cdot\left(\mathbf{v}\cdot\partial\right)\partial v^a\right) \ , \end{aligned} \tag{3.3.24}$$

$$\partial_b K^{ba} = c_1\left(\partial\cdot\partial\right)v^a + (c_2 + c_3)\partial^a\left(\partial\cdot\mathbf{v}\right) + c_4\left(\left(\partial\cdot\mathbf{v}\right)\frac{\mathrm{d}v^a}{\mathrm{d}t} + \frac{\mathrm{d}^2 v^a}{\mathrm{d}t^2} + \mathbf{v}\cdot\frac{\mathrm{d}\partial v^a}{\mathrm{d}t}\right) \ . \tag{3.3.25}$$

We used $v^a\partial_a = \mathbf{v}\cdot\partial = \mathrm{d}/\mathrm{d}t$ which corresponds to the derivative along the wordline of æther and $\partial_a v^a = \partial\cdot\mathbf{v}$.

In the limit of a large number of particles at random velocities, we can assimilate the particle system behaves like continuous matter at density ρ, kinetic pressure field P and velocity u^a, where u^a is the effective velocity that each particle would have. Using the same idea, we define $F_{eff}(\gamma, P) \equiv F(\gamma_p)$, which represents the effective coupling of each particle with æther. The presence of P in this effective coupling is to highlight the fluctuations of the real coupling of each particle around this effective value, but in the rest we will neglect these effects. According to the procedure of ([7], page 43), the action of the matter (3.3.22) is rewritten as follows

$$S_M = -\int \mathrm{d}^4x(\rho - 3P)F_{eff}(\gamma)\ , \tag{3.3.26}$$

where $\gamma = -v_a u^a$. Hence the equation of motion of a body (fluid) reads

$$\partial_b K^{ba} = c_4\frac{\mathrm{d}v_c}{\mathrm{d}t}\partial^a v^c + \lambda v^a - 8\pi(\rho-3P)Gu^aF'_{eff}(\gamma)\ . \tag{3.3.27}$$

Varying λ gives the constraint $\eta_{ab}v^av^b = -1$. Equation (3.3.27) can be used to eliminate λ, giving

$$\lambda = -\left(v^a\partial_b K^b{}_a - c_4\dot{v}_c\dot{v}^c + 8\pi(\rho-3P)G\gamma F'_{eff}(\gamma)\right)\ , \tag{3.3.28}$$

where $\dot{v}^c = \mathrm{d}v^c/\mathrm{d}t$. Equation (3.3.28) leads to the equation motion

$$\left(\eta^{ac} + v^av^c\right)\partial_b K^b{}_c = c_4\dot{v}_c\left(\partial^a v^c + v^a\dot{v}^c\right) - 8\pi(\rho-3P)GF'_{eff}(\gamma)\left(u^a + \gamma v^a\right)\ . \tag{3.3.29}$$

In order to clarify the meaning of the equation of motion (3.3.29), we will simplify it, by making it easier to understand. Suppose that $c_4 = 0$

$$\left(\eta^{ac} + v^av^c\right)\left(c_1\Box v_c + (c_2+c_3)\partial_c\left(\partial\cdot\mathbf{v}\right)\right) = -8\pi(\rho-3P)GF'_{eff}(\gamma)\left(u^a+\gamma v^a\right)\ , \tag{3.3.30}$$

where $\Box = \partial\cdot\partial$. Moreover, assuming that the speeds are low enough to make an approximation in the non-relativist case, i. e. $u \ll 1$, $v \ll 1$ and $P \ll \rho$, we obtain

$$c_1\Box v^a + (c_2+c_3)\eta^{ac}\partial_c\left(\partial\cdot\mathbf{v}\right) = -8\pi\rho GF'_{eff}(\gamma)u^a\ . \tag{3.3.31}$$

According to our expectations from the analysis made by the equation (3.3.2), which is to have a derivative of a quadratic form at the speed of æther (Δv^2), we notice that the equation (3.3.31) is linear in v^a.

3.4 Discussion

It has been shown in this chapter that the coupling between matter (particle) and æther within the Newtonian limit, suggests that matter should cool down the temperature of æther or depressurizes æther. Note that in the case of spherical symmetry, the diffusion phenomenon will create an accumulation of æther around the material. This scenario is not appealing because it would prevent the existence of a stationary regime.

The relativistic approach that we have tried to explore using the action of the æther field of Einstein-aether theory does not offer a translucent interpretation, which suggests that this action may not be appropriate to describe our model as expected in the Newtonian limit.

4. Conclusion

We now conclude with some remarks on the meaning and possible implications of these results. First of all, to summarize, we argued that Newton's gravitation can be described as the flow of æther created by absorption of æther. The flow of æther depends on the mass of matter. The way in which matter undergoes æther may also depend on its shape. Nevertheless, the absorption model is able to describe the problem of two bodies. In addition to Newton's gravitation force, we have shown the existence of friction forces on two-body problem. Since the flow rate of æther can be higher than the speed of light, it seemed necessary to account for the relativistic effects which led us to the thermal model. This model is inspired from Einstein-æther theory. The behaviour of matter coupled to æther at the Newtonian limit, suggests an analogy with gravitation. We have shown that for a stationary and irrotational flow, assuming that it is a perfect fluid, the temperature change leads to a convective flow which behaves like gravity.

In addition to the fact that we were able to recover some aspects of gravitation after some hypotheses, i.e. by turning some parameters of the problem. We can conclude by saying that the analogies between hydrodynamics and gravity may have a physical meaning. Although, the models presented in this work still need to be improved, we can expect to find a model that will explain gravity more deeply. We suggest that thermodynamics will contribute to this analogy.

Our future perspectives would be to further study the thermal model at the relativistic level, in order to find the right æther's action $S_{æ}$, which will reproduce the expected effects as within the Newtonian limit. If we find such an action, we will explore its properties and examine its ability to describe phenomena such as the deviation of light, the precession of Mercury's perihelion, the existence of black holes and gravitational waves.

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